| Surname: | Forename(s): |
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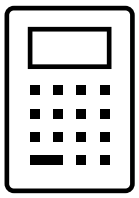
# A Level Mathematics – Pure Maths

# Paper 2



## Practice Set D

## ANSWERS

**Materials**

/50

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

**Instructions**

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

# A Level (Pure) Formulae Sheet

## Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

## Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

## Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

## Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

## Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

## Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

| $f(x)$ | $f'(x)$ |
|---|---|
| $\tan kx$ | $k \sec^2 kx$ |
| $\sec kx$ | $k \sec kx \tan kx$ |
| $\cot kx$ | $-k \operatorname{cosec}^2 kx$ |
| $\operatorname{cosec} kx$ | $-k \operatorname{cosec} kx \cot kx$ |
| $\frac{f(x)}{g(x)}$ | $\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ |

## Integration (+ constant)

| $f(x)$ | $\int f(x)\,dx$ |
|---|---|
| $\sec^2 kx$ | $\frac{1}{k}\tan kx$ |
| $\tan kx$ | $\frac{1}{k}\ln\lvert\sec kx\rvert$ |
| $\cot kx$ | $\frac{1}{k}\ln\lvert\sin kx\rvert$ |
| $\operatorname{cosec} kx$ | $-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$ |
| $\sec kx$ | $\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$ |

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

# A Level (Pure) Formulae Sheet continued

## Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$ : $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

## Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

where $\theta$ is measured in radians

1 (a) Simplify $\frac{8}{(3-\sqrt{5})^2}$

Give your answer in the form $a + b\sqrt{5}$

$\hookrightarrow \frac{8}{(3-\sqrt{5})^2}$ ← $(3-\sqrt{5})^2 = (3-\sqrt{5})(3-\sqrt{5})$

$= 9 - 3\sqrt{5} - 3\sqrt{5} + 5$

$= 14 - 6\sqrt{5}$

$\hookrightarrow \frac{8}{(14-6\sqrt{5})} \times \frac{(14+6\sqrt{5})}{(14+6\sqrt{5})}$

$\hookrightarrow \frac{112 + 48\sqrt{5}}{196 + 84\sqrt{5} - 84\sqrt{5} - 180}$

$\hookrightarrow \frac{112 + 48\sqrt{5}}{16}$

$\hookrightarrow 7 + 3\sqrt{5}$

$7 + 3\sqrt{5}$

(4)

1 (b) Simplify $\left(1296^{\frac{1}{2}} - 27^{\frac{2}{3}}\right)^{-\frac{1}{3}}$

Give your answer as an exact fraction in its simplest form.

$\left(1296^{\frac{1}{2}} - 27^{\frac{2}{3}}\right)^{-\frac{1}{3}}$

$\hookrightarrow (36 - 9)^{-\frac{1}{3}}$

$\hookrightarrow (27)^{-\frac{1}{3}}$

$\hookrightarrow \left(\frac{1}{27}\right)^{\frac{1}{3}}$

$\hookrightarrow \frac{1}{3}$

$\frac{1}{3}$

(3)

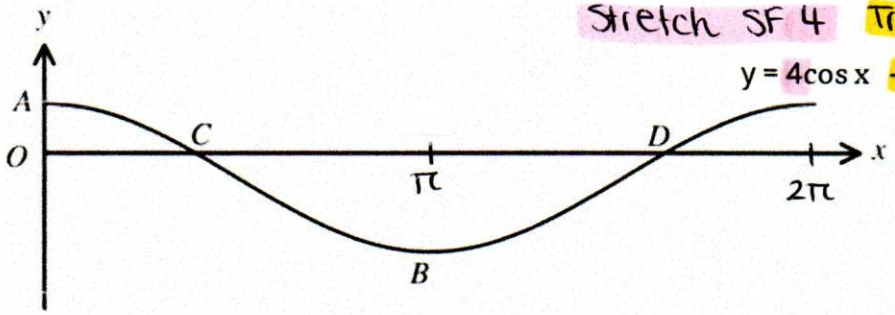
(Total for question 1 is 7 marks)

2 Here is the graph of $y = 4\cos x - 1$ $\quad 0 \leq x \leq 2\pi$

Transformation in y

Stretch SF 4 Translation −1

$y = 4\cos x - 1$




The graph intersects the coordinate axes at points A, C and D.
Point B is the first minimum point of this graph for which $x > 0$

(a) State the coordinates of point A and point B.

A( 0 , 3 )

B( π , −5 )

(2)

(b) State the coordinates of point C and point D.
Give your answers correct to 3 significant figures.

$4\cos x - 1 = 0$

$4\cos x = 1$

$\cos x = \frac{1}{4}$

$x = 1.318116 \approx 1.32$

or

$x = 2\pi - 1.318116 = 4.965069 \approx 4.97$

C( 1.32 , 0 )

D( 4.97 , 0 )

(4)

(Total for question 2 is 6 marks)

3 The fourth and fifth terms of a geometric progression are 320 and 128, respectively.

(a) Find the common ratio of this progression.

$t_n = ar^{n-1}$

$t_4 = ar^3 = 320$

$t_5 = ar^4 = 128$

$\hookrightarrow \frac{ar^5}{ar^4} = \frac{128}{320}$

$\hookrightarrow r = \frac{2}{5}$

$\frac{2}{5}$ OR 0.4

(1)

(b) Find the first term of this progression.

$ar^3 = 320$

$a\left(\frac{2}{5}\right)^3 = 320$

$a = 320 \div \left(\frac{2}{5}\right)^3$

$a = 5000$

5000

(2)

(b) Find the sum to infinity of this progression.

$S_\infty = \frac{a}{1-r}$

$= \frac{5000}{1-\frac{2}{5}}$

$= \frac{25000}{3}$ OR 8333.33...

$\frac{25000}{3}$

(2)

(Total for question 3 is 5 marks)

4 $f(x) = 3x^3 + 13x^2 - 9x + 5$

(a) Show that ( x + 5 ) is a factor of f(x)

If $(x+5)$ is a factor of $f(x)$, $f(-5) = 0$

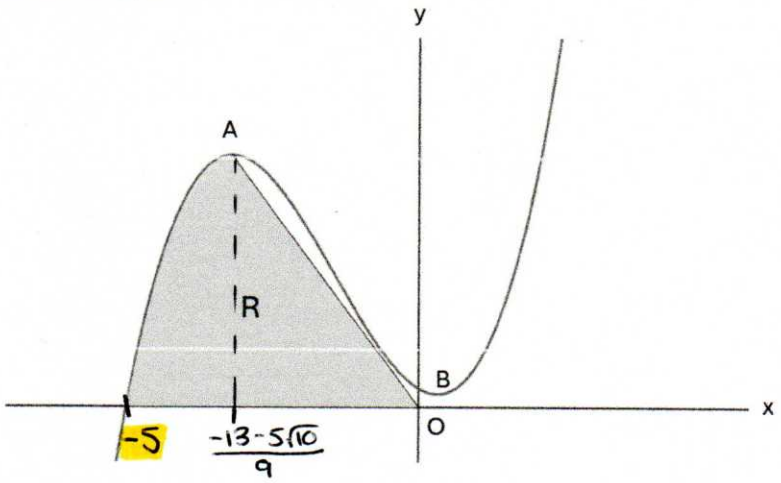
$f(-5) = 3(-5)^3 + 13(-5)^2 - 9(-5) + 5$

$= 0$

$\therefore (x+5)$ is a factor of $f(x)$. (2)

(b) The figure below shows the curve with equation y = f(x)

A and B are stationary points of the curve.




Find the ~~exact~~ coordinates of point A and point B

$y = 3x^3 + 13x^2 - 9x + 5$

$\frac{dy}{dx} = 9x^2 + 26x - 9 = 0$

$x = \frac{-13+5\sqrt{10}}{9} = 0.312376\ldots$ $y = 3.548583\ldots$

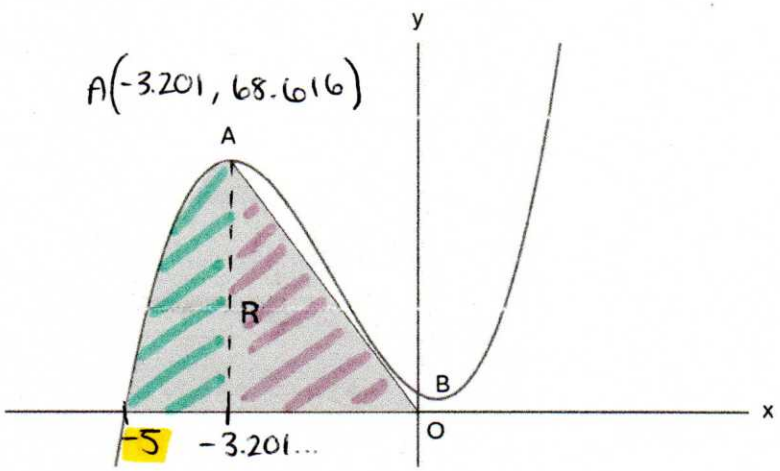
$x = \frac{-13-5\sqrt{10}}{9} = -3.201265\ldots$ $y = 68.616025\ldots$

A( -3.201 , 68.616 )

B( 0.312 , 3.549 )

(5)

4 (continued)




The region R is bounded by the x axis, the curve and the straight line OA.
Determine the area of R.

$R = \int_{-5}^{-3.201} f(x)\,dx$ + 

$\int_{-5}^{-3.201} 3x^3 + 13x^2 - 9x + 5\,dx$

$\hookrightarrow \left[\frac{3x^4}{4} + \frac{13x^3}{3} - \frac{9x^2}{2} + 5x\right]_{-5}^{-3.201}$

$\hookrightarrow \left[\frac{3(-3.201)^4}{4} + \frac{13(-3.201)^3}{3} - \frac{9(-3.201)^2}{2} + 5(-3.201)\right] - \left[\frac{3(-5)^4}{4} + \frac{13(-5)^3}{3} - \frac{9(-5)^2}{2} + 5(-5)\right]$

$\hookrightarrow -125.5182659... - -210.41666...$

$\hookrightarrow 84.8984...$

 $= \frac{b \times h}{2} = \frac{3.201... \times 68.616...}{2} = 109.829...$

$84.8984... + 109.829... = 194.727...$

194.7 units²

(8)

(Total for question 4 is 15 marks)

5 Solve $\log_3 8 - 3\log_3 t = 3$

$\log_3 8 - 3\log_3 t = 3$

$\log_3 8 - \log_3 t^3 = \log_3 27$

$\log_3 \left(\frac{8}{t^3}\right) = \log_3 27$

↳ $\frac{8}{t^3} = 27$

↳ $8 = 27t^3$

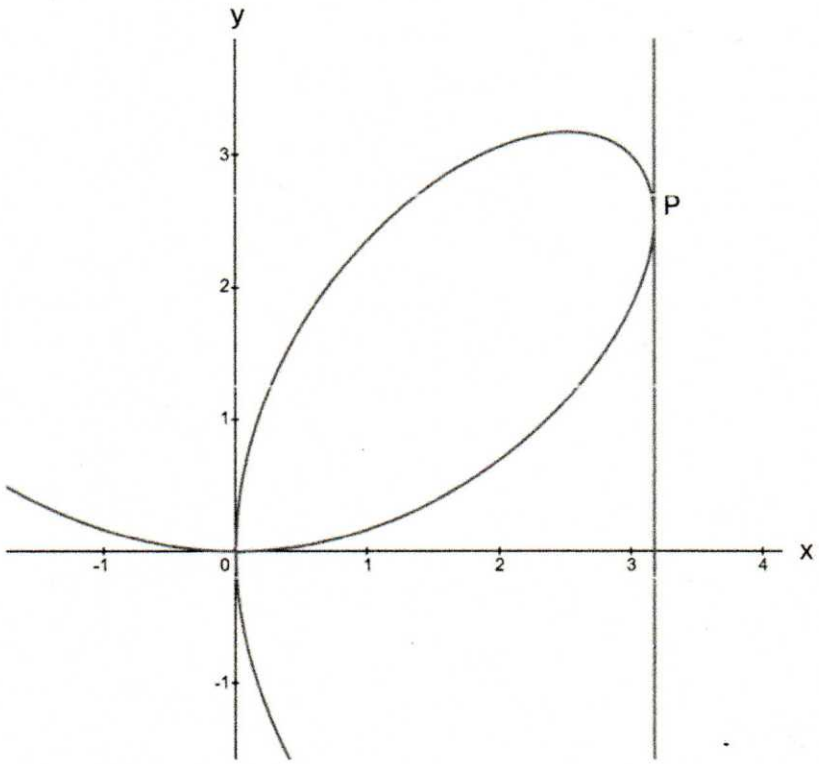
↳ $\frac{8}{27} = t^3$

↳ $\frac{2}{3} = t$

t = $\frac{2}{3}$

(Total for question 5 is 5 marks)

6 The diagram shows curve with equation $x^3 - 6xy + y^3 = 0$
The tangent to the curve at point P is parallel to the y axis.
Find an equation for the tangent at point P.




$x^3 - 6xy + y^3 = 0$

$\hookrightarrow 3x^2 - 6y - 6x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\hookrightarrow 3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$

$\hookrightarrow \frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$

$\hookrightarrow \frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$

$\hookrightarrow \frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

$u = -6x \quad v = y$
$u' = -6 \quad v' = \frac{dy}{dx}$
$\hookrightarrow u'v + uv'$
$\hookrightarrow -6y - 6x\frac{dy}{dx}$

Parallel to x - numerator = 0
Parallel to y - denominator = 0

6 (continued)

$x^3 - 6xy + y^3 = 0$

$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

$y^2 - 2x = 0$

$y^2 = 2x$

$\frac{y^2}{2} = x$

$\left(\frac{y^2}{2}\right)^3 - 6\left(\frac{y^2}{2}\right)y + y^3 = 0$

$\frac{1}{8}y^6 - 3y^3 + y^3 = 0$

$\frac{1}{8}y^6 - 2y^3 = 0$

$y^3\left(\frac{1}{8}y^3 - 2\right) = 0$

$y^3 = 0$

$y = 0$

$\frac{1}{8}y^3 - 2 = 0$

$\frac{1}{8}y^3 = 2$

$y^3 = 16$

$y = \sqrt[3]{16} = 2^{\frac{4}{3}}$

$x = \frac{\left(2^{\frac{4}{3}}\right)^2}{2}$

$x = \frac{2^{\frac{8}{3}}}{2}$

$x = 2^{\frac{5}{3}}$

$x = 2^{\frac{5}{3}}$

(Total for question 6 is 12 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS