Surname: | Forename(s):

A Level Mathematics – Pure Maths

Paper 1

Practice Set B

MyStudy success

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k\operatorname{cosec} kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

Integration (+ constant)

$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

A Level (Pure) Formulae Sheet continued

Numerical Methods

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

where θ is measured in radians

1 Simplify fully

(a) $\sqrt{80x} \times \sqrt{5x^5}$

(2)

(b) $(27y^6)^{2/3} \times \left(\frac{4}{y^2}\right)^{1/2}$

(2)

(c) $9 \times 32^{2w} - 5 \times 4^{5w}$

(4)

(Total for question 1 is 8 marks)

2 A circle has equation

$$x^2 + y^2 + 8x - 12y - 48 = 0$$

(a) Find

(i) The radius of the circle

(ii) The coordinates of the centre of the circle

Radius = ____________________

Centre (_________ , _________)

(3)

(b) Given that C is the point on the circle that is closest to the origin O.

Find the exact length of OC

____

(2)

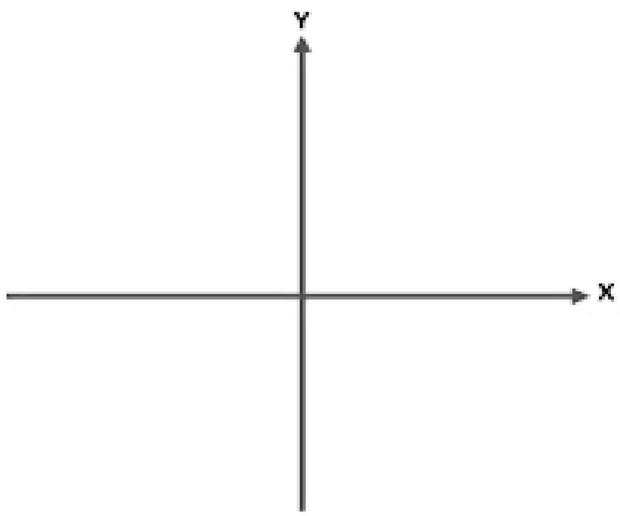
(Total for question 2 is 5 marks)

3 The polynomial f(x) is defined as $2x^3 + x^2 - 43x - 60$.

(i) Given that $(x - 5)$ is a factor of f(x), express f(x) in the form $(x - 5) (x + a) (bx + c)$, where a, b and c are integers to be found.

(3)

(ii) Sketch the graph of $y = f(x)$, clearly labelling the coordinates of any points of intersection with the axes.



(2)

3 (continued)

(iii) Solve the inequality $f(x) \leq 0$. Give your answer in set notation.

(2)

(iv) The graph of $y = f(x)$ is transformed by a translation 2 units to the left.
Find the equation of the transformed graph.

(2)

(Total for question 3 is 9 marks)

4 A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k\,(a_n + 2)}{a_n}$$

(a) Given that

- k is a constant
- $a_1 = 2$
- the sequence is a periodic sequence of order 3

Show that $k^2 + k - 2 = 0$

(3)

4 (continued)

(b) For this sequence, explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{65} a_r$$

(3)

(Total for question 4 is 7 marks)

5 Find the exact value of x for which

$$\log_5(5x) = \log_5(2x + 3) - 2$$

Give your answer in its simplest form.

x = ______________________

(Total for question 5 is 4 marks)

6 (a) Prove that

$$\frac{1-\cos 2\theta + \sin 2\theta}{1+\cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

where $1 + \cos 2\theta + \sin 2\theta \neq 0$

(4)

(question 5 continued)

6 b) Hence solve, for 0 < x < 180°

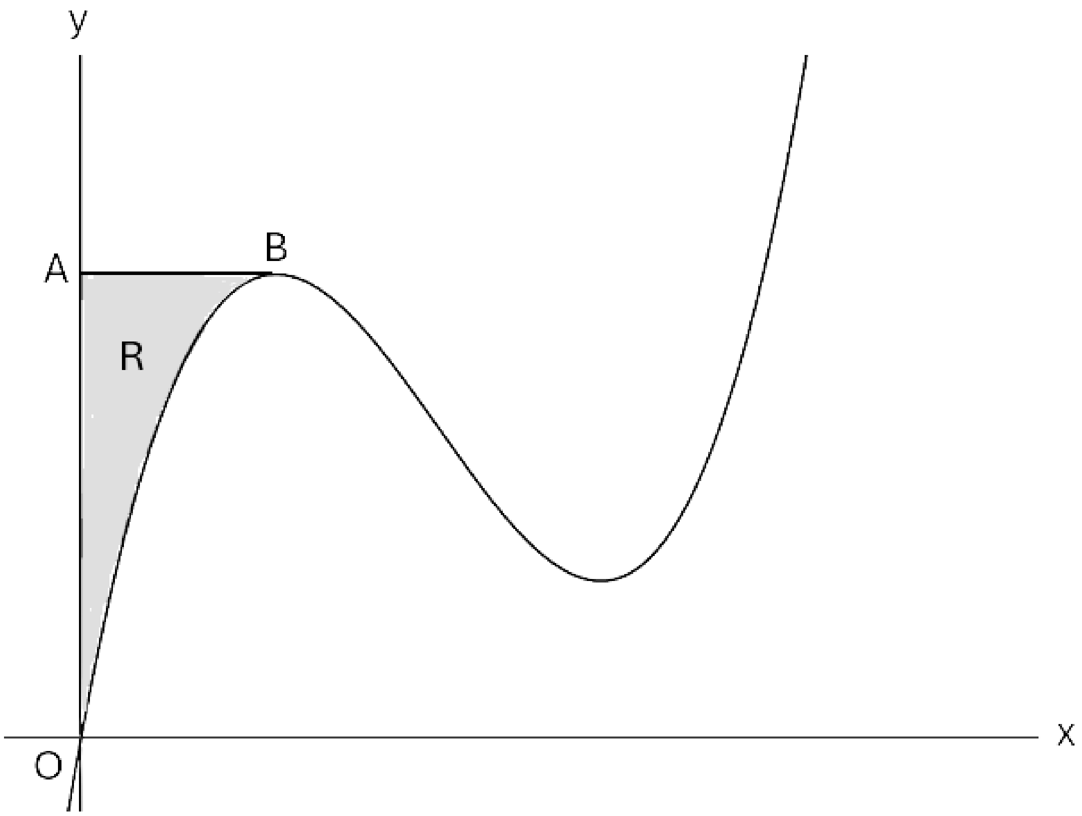
$$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x} = 5\sin 2x$$

Give your answers to one decimal place where appropriate.

(4)

(Total for question 6 is 8 marks)

7 The curve with equation $y = x^3 - 11x^2 + kx$ is shown below.
k is a constant.



Point B is the maximum turning point of the curve.

(a) Given that the x coordinate at point B is 2, show that k = 32

(3)

7 (continued)

The region R is bounded by the y axis, the horizontal line AB, and the curve with equation $y = x^3 - 11x^2 + 32x$

(b) Calculate the exact area of the region R.

You must show your working.

______________________ units^2

(6)

(Total for question 7 is 9 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS