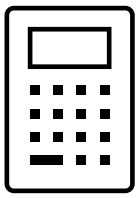
Surname:	Forename(s):

A Level Mathematics – Pure Maths

Paper 1



Practice Set C

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec kx$	$k \sec kx \tan kx$
$\cot kx$	$-k \operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k \operatorname{cosec} kx \cot kx$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

$f(x)$	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

A Level (Pure) Formulae Sheet continued

Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

where θ is measured in radians

1 Differentiate the following expressions with respect to x.

(a) $y = (2 - x^2)^8$

$\frac{dy}{dx} =$ _____

(2)

(b) $y = x^4 \sin 4x$

$\frac{dy}{dx} =$ _____

(3)

1 (continued)

(c) $y = \frac{2x}{x^3 + 3}$

$\frac{dy}{dx} =$ _____

(3)

(Total for question 1 is 8 marks)

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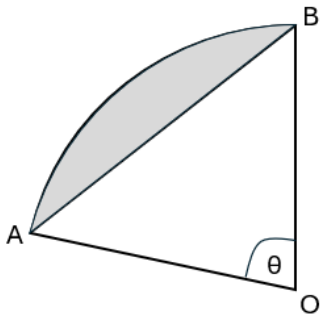
2 Here is sector OAB with centre O.

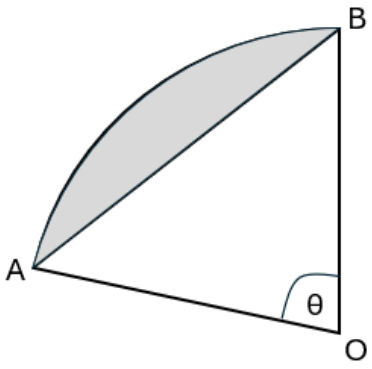


The radius of the sector is 52 mm.
The length of the chord AB is 66mm.
The angle θ is measured in radians.

(a) Show that $\theta \approx 1.375$ C

(3)

2 (continued)



(b) Using the approximation $\theta \approx 1.375$ [C], calculate the perimeter of sector OAB.

_____ mm

(3)

(c) Using the approximation $\theta \approx 1.375$ [C], calculate the area of the shaded segment.

_____ mm^2

(3)

(Total for question 2 is 9 marks)

3 £43,875 is divided into three shares, so that the three shared form a geometric progression.
Given that the value of the smallest share is £4500, calculate the value of the largest share.

£ _____

(Total for question 3 is 6 marks)

4

$$f(x) = x^3 - x^2 - 5x + 5$$

(a) Show that (x-1) is a factor of f(x)

(2)

(b) Solve f(x) = 0, giving the exact values for your answer.

(4)

4 (continued)

$$f(x) = x^3 - x^2 - 5x + 5$$

(c) Hence, or otherwise, solve the trigonometric equation

$$\tan^3\theta - \tan^2\theta - 5\tan\theta + 5 = 0$$

for $0 \leq \theta \leq 360$

(5)

(Total for question 4 is 11 marks)

5 (a) Solve $\log_x(9) = 2$

x = _____

(2)

(b) Solve $\log_2(4y + 1) - 2\log_2(y + 2) = 2 - \log_2(y + 1)$

y = _____

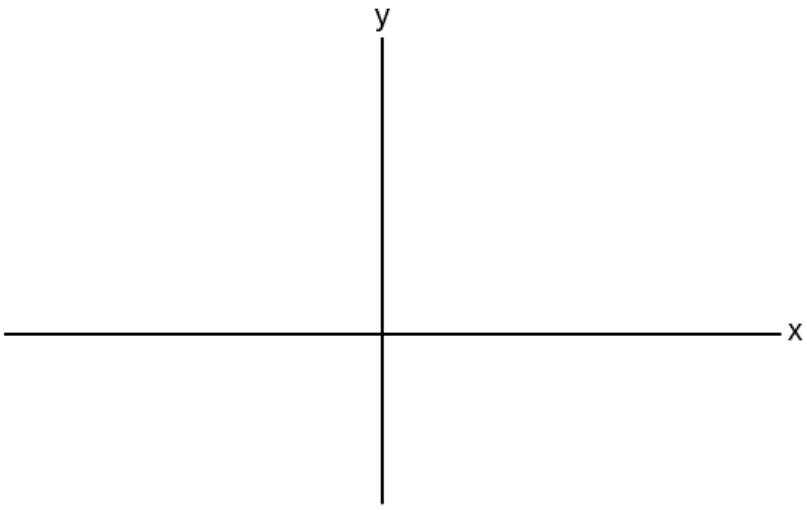
(6)

(Total for question 5 is 8 marks)

6

$$g(x) = e^{3x} - 1 \qquad x \in \mathbb{R}$$
$$h(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of hg(x)



(5)

(b) Solve the equation hg(x) = 1

x = _____

(3)

(Total for question 6 is 8 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS