Surname: | Forename(s):

A Level Mathematics – Pure Maths

Paper 1

Practice Set C

ANSWERS

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n - 1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n - r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\,\mathrm{cosec}^2\, kx$
$\mathrm{cosec}\, kx$	$-k\,\mathrm{cosec}\, kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

Integration	**(+ constant)**
$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\mathrm{cosec}\, kx$	$-\frac{1}{k}\ln\lvert\mathrm{cosec}\, kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

A Level (Pure) Formulae Sheet continued

Numerical Methods

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$

$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

Small angle approximations

$\sin \theta \approx \theta$

$\cos \theta \approx 1 - \frac{\theta^2}{2}$

$\tan \theta \approx \theta$

where θ is measured in radians

1 Differentiate the following expressions with respect to x.

(a) $y = (2 - x^2)^8$

$y = (2 - x^2)^8$ (chain rule)

$\frac{dy}{dx} = 8(2 - x^2)^7(-2x)$

$\frac{dy}{dx} = -16x(2 - x^2)^7$

$\frac{dy}{dx} = \underline{-16x(2 - x^2)^7}$

(2)

(b) $y = x^4 \sin 4x$

$y = x^4 \sin 4x$ (product rule)

$y = uv$

$u = x^4$ $v = \sin 4x$ $\frac{dy}{dx} = u'v + uv'$

$u' = 4x^3$ $v' = 4\cos 4x$

$\frac{dy}{dx} = 4x^3 \sin 4x + 4x^4 \cos 4x$

$\frac{dy}{dx} = 4x^3(\sin 4x + x\cos 4x)$

$\frac{dy}{dx} = \underline{4x^3(\sin 4x + x\cos 4x)}$

(3)

1 (continued)

(c) $y = \frac{2x}{x^3 + 3}$

(Quotient rule)

$y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$

$u = 2x \qquad v = x^3 + 3$

$u' = 2 \qquad v' = 3x^2$

$\frac{dy}{dx} = \frac{2(x^3+3) - 2x(3x^2)}{(x^3+3)^2}$

$= \frac{2x^3 + 6 - 6x^3}{(x^3+3)^2}$

$= \frac{6 - 4x^3}{(x^3+3)^2}$

$\frac{dy}{dx} = \frac{6 - 4x^3}{(x^3+3)^2}$

(3)

(Total for question 1 is 8 marks)

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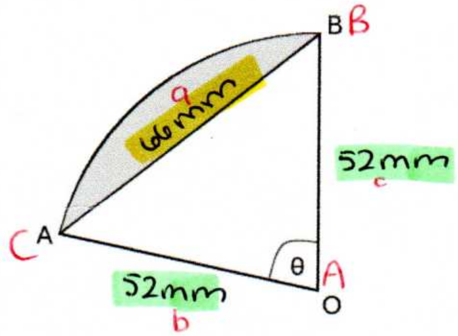
2 Here is sector OAB with centre O.



The radius of the sector is 52 mm.
The length of the chord AB is 66mm.
The angle θ is measured in radians.

(a) Show that θ ≈ 1.375 ^{c}

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \frac{(52)^2 + (52)^2 - (66)^2}{2(52)(52)}$

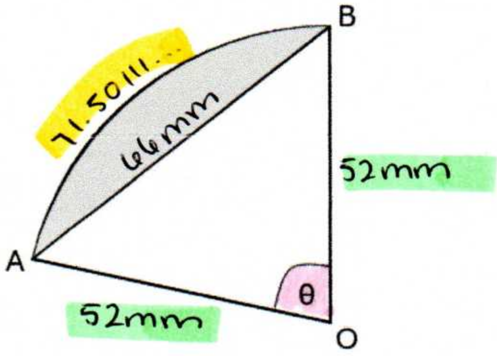
$\cos\theta = 0.1945266272$

$\theta = \cos^{-1}(0.194\ldots) = 1.375021491$

≈ 1.375

(3)

2 (continued)



(b) Using the approximation $\theta \approx 1.375^c$, calculate the perimeter of sector OAB.

Arc length $= r\theta$

$= 52(1.375) = 71.50111...$

$71.50111... + 52 + 52 = 175.50111...$

175.5 mm

(3)

(c) Using the approximation $\theta \approx 1.375^c$, calculate the area of the shaded segment.

$= \left(\frac{1}{2}r^2\theta\right) - \left(\frac{1}{2}ab\sin c\right)$

$= \left(\frac{1}{2}(52)^2(1.375)\right) - \left(\frac{1}{2}(52)(52)\sin(1.375)\right)$

$= 532.8325869...$

532.8 mm^2

(3)

(Total for question 2 is 9 marks)

3 £43,875 is divided into three shares, so that the three shared form a geometric progression.
Given that the value of the smallest share is £4500, calculate the value of the largest share.

1 4500 2 $4500r$ 3 $4500r^2$

$4500 + 4500r + 4500r^2 = 43,875$

$-43,875$ $-43,875$

$4500r^2 + 4500r - 39375 = 0$

$\div 75$

$60r^2 + 60r - 525 = 0$

$\div 15$

$4r^2 + 4r - 35 = 0$

$(2r + 7)(2r - 5) = 0$

$\cancel{r = -\frac{7}{2}}$ or $r = \frac{5}{2}$

invalid
$r > 0$

1 4500

2 $4500r = 4500\left(\frac{5}{2}\right) = 11,250$

3 $4500r^2 = 4500\left(\frac{5}{2}\right)^2 = 28,125$

£ 28,125

(Total for question 3 is 6 marks)

4

$$f(x) = x^3 - x^2 - 5x + 5$$

(a) Show that (x-1) is a factor of f(x)

If $(x-1)$ is a factor of $f(x)$, then $f(1) = 0$

$f(1) = (1)^3 - (1)^2 - 5(1) + 5 = 0$

$\therefore (x-1)$ is a factor of $f(x)$

(2)

(b) Solve f(x) = 0, giving the exact values for your answer.

$$
\begin{array}{r|l}
 & x^2 \qquad\quad - 5 \\
\hline
(x-1) & x^3 - x^2 - 5x + 5 \\
 & -(x^3 - x^2) \\
\hline
 & \qquad\quad -5x + 5 \\
 & - \qquad (-5x + 5) \\
\hline
 & \qquad\qquad\quad 0
\end{array}
$$

$f(x) = (x-1)(x^2 - 5) = 0$

$x = 1$

$x^2 - 5 = 0$

$+5 \quad +5$

$x^2 = 5$

$\sqrt{\ } \quad \sqrt{\ }$

$x = \pm\sqrt{5}$

$x = 1,\ x = \sqrt{5},\ x = -\sqrt{5}$

(4)

4 (continued)

$$f(x) = x^3 - x^2 - 5x + 5$$

$x = \tan\theta$

(c) Hence, or otherwise, solve the trigonometric equation

$$\tan^3\theta - \tan^2\theta - 5\tan\theta + 5 = 0$$

for $0 \leq \theta \leq 360$

$x = 1$
$\tan\theta = 1$
$\theta = 45$
$\theta = 225$ $(45 + 180)$

$x = \sqrt{5}$
$\tan\theta = \sqrt{5}$
$\theta = 65.905\ldots$
$\theta = 245.905\ldots$
$(65.905 + 180)$

$x = -\sqrt{5}$
$\tan\theta = -\sqrt{5}$
$\theta = -65.905\ldots$ ✗ invalid
$\theta = 114.095\ldots$
$(-65.905 + 180)$
$\theta = 294.095\ldots$
$(114.095 + 180)$

$\theta = 45°, 65.9°, 114.1°, 225°, 245.9°, 294.1°$

(5)

(Total for question 4 is 11 marks)

5 (a) Solve $\log_x(9) = 2$

$\log_x 9 = 2$

$9 = x^2$

$\sqrt{\ }$ $\sqrt{\ }$

$3 = x$

$(x > 0)$

x = 3

(2)

(b) Solve $\log_2(4y + 1) - 2\log_2(y + 2) = 2 - \log_2(y + 1)$

$\log_2(4y+1) - 2\log_2(y+2) = 2 - \log_2(y+1)$

$\hookrightarrow \log_2(4y+1) - \log_2(y+2)^2 = \log_2 4 - \log_2(y+1)$

$\hookrightarrow \log_2\left(\frac{4y+1}{(y+2)^2}\right) = \log_2\left(\frac{4}{y+1}\right)$

$\hookrightarrow \frac{4y+1}{(y+2)^2} = \frac{4}{y+1}$

$\hookrightarrow (4y+1)(y+1) = 4(y+2)(y+2)$

$\hookrightarrow 4y^2 + 4y + y + 1 = 4(y^2 + 2y + 2y + 4)$

$\hookrightarrow 4y^2 + 5y + 1 = 4(y^2 + 4y + 4)$

$\hookrightarrow \cancel{4y^2} + 5y + 1 = \cancel{4y^2} + 16y + 16$

$\hookrightarrow 5y + 1 = 16y + 16$

$-5y$ $-5y$

$\hookrightarrow 1 = 11y + 16$

-16 -16

$\hookrightarrow -15 = 11y$

$\div 11$ $\div 11$

$\hookrightarrow -\frac{15}{11} = y$

y = $-\frac{15}{11}$

(6)

(Total for question 5 is 8 marks)

6

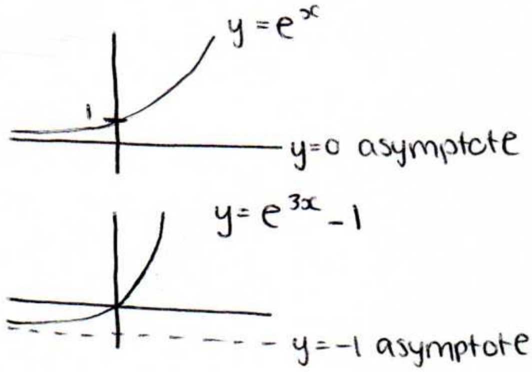
enlargement in x axis scale factor $\frac{1}{3}$ — translation 1 down

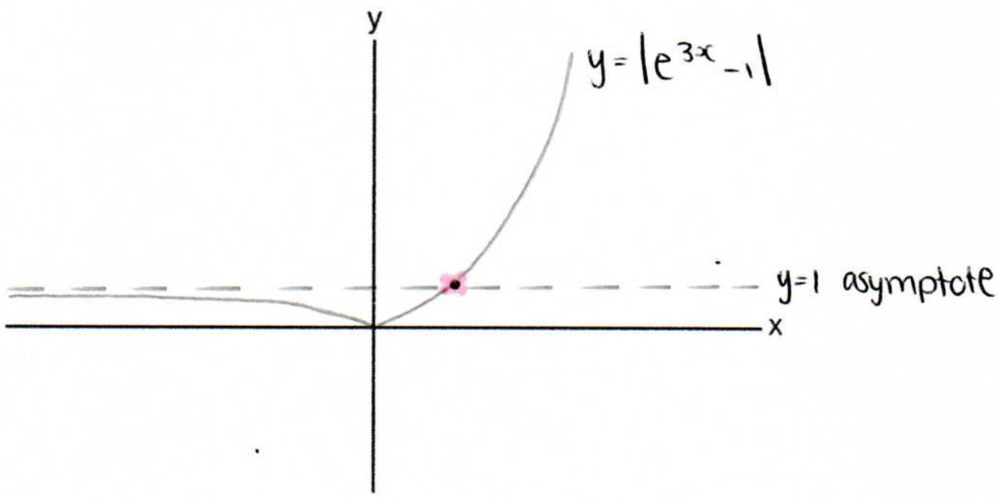
$$g(x) = e^{3x} - 1 \qquad x \in \mathbb{R}$$

$$h(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of hg(x)

$h(gx) = |e^{3x} - 1|$





(5)

(b) Solve the equation hg(x) = 1

$|e^{3x} - 1| = 1$

↳ $e^{3x} - 1 = 1$ (+1) (+1)

↳ $e^{3x} = 2$

↳ $3x = \ln 2$ (÷3) (÷3)

↳ $x = \frac{\ln 2}{3}$

x = $\frac{\ln 2}{3}$

(3)

(Total for question 6 is 8 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS