Surname: | Forename(s):

A Level Mathematics – Pure Maths

Paper 2

Practice Set B

ANSWERS

MyStudy success

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec kx$	$k \sec kx \tan kx$
$\cot kx$	$-k \operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k \operatorname{cosec} kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

Integration	**(+ constant)**
$f(x)$	$\int f(x)\, dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

Small angle approximations

$\sin\theta \approx \theta$

$\cos\theta \approx 1 - \dfrac{\theta^2}{2}$

$\tan\theta \approx \theta$

where θ is measured in radians

1 A rectangle has an area of $5 + 7\sqrt{5}$ cm^2
The base of the rectangle is $3 + 2\sqrt{5}$ cm
Work out the height of the rectangle.
Give your answer as an exact surd in its simplest form.

Area = base × height

height = $\dfrac{\text{Area}}{\text{base}}$

$$= \frac{(5+7\sqrt{5})}{(3+2\sqrt{5})} \times \frac{(3-2\sqrt{5})}{(3-2\sqrt{5})}$$

$$= \frac{15 - 10\sqrt{5} + 21\sqrt{5} - 70}{9 - 6\sqrt{5} + 6\sqrt{5} - 20}$$

$$= \frac{-55 + 11\sqrt{5}}{-11}$$

$$= 5 - \sqrt{5}$$

$5 - \sqrt{5}$

(Total for question 1 is 5 marks)

2 The polynomial f(x) is defined as

$$f(x) = x^3 + ax^2 + bx + 15$$

(a) Given that:

- $(x-1)$ is a factor of f(x) $\longrightarrow f(1) = 0$
- f(x) divided by (x+1) has a remainder of 24. $\longrightarrow f(-1) = 24$

Find the values of constants a and b.

$f(1) = (1)^3 + a(1)^2 + b(1) + 15$
$= a + b + 16$

$a + b + 16 = 0$
$a + b = -16$

$f(-1) = (-1)^3 + a(-1)^2 + b(-1) + 15$
$= a - b + 14$

$a - b + 14 = 24$
$a - b = 10$

$+$
$a + b = -16$
$a - b = 10$
$2a = -6$
$a = -3$

$b = -16 - a$
$b = -16 - (-3)$
$b = -13$

a = -3

b = -13

(6)

(b) Hence, or otherwise, solve the equation f(x) = 0

$f(x) = x^3 - 3x^2 - 13x + 15$
$(x-1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 - 2x - 15 \\ \hline x-1 & x^3 - 3x^2 - 13x + 15 \\ & -(x^3 - x^2) \\ \hline & -2x^2 - 13x + 15 \\ & -(-2x^2 + 2x) \\ \hline & -15x + 15 \\ & -(-15x + 15) \\ \hline & 0 \end{array}$$

$f(x) = (x-1)(x^2 - 2x - 15)$
$= (x-1)(x-5)(x+3)$

$x = 1 \quad x = 5 \quad x = -3$

$\boxed{x = 1, x = 5, x = -3}$

(4)

(Total for question 2 is 10 marks)

3 Solve the following exponential equations.
Give your answers to 3 significant figures.

(a) $2^{3x-2} = 3^{23}$

$3x - 2 = \log_2 3^{23}$

$3x - 2 = 23\log_2 3$

$3x = (23\log_2 3) + 2$

$x = \dfrac{(23\log_2 3)+2}{3}$

$x = 12.81804584\ldots$

x = 12.8

(3)

(b) $5^{y+1} = \frac{10}{5^y}$

$5^{y+1} \times 5^y = 10$

$5^{2y+1} = 10$

$2y+1 = \log_5 10$

$2y = (\log_5 10) - 1$

$y = \dfrac{(\log_5 10) - 1}{2}$

$y = 0.215338279$

y = 0.215

(5)

(Total for question 3 is 8 marks)

4 Solve the following trigonometric equation for $0 < \theta \leq 180$

$\hookrightarrow 0 < 2\theta \leq 360$

$$\frac{1}{3}\tan 2\theta - 2\sin 2\theta = 0$$

$\tan x = \dfrac{\sin x}{\cos x}$

$\frac{1}{3}\tan 2\theta - 2\sin 2\theta = 0$

$\hookrightarrow \dfrac{\sin 2\theta}{3\cos 2\theta} - 2\sin 2\theta = 0$

$\times 3\cos 2\theta$

$\hookrightarrow \sin 2\theta - 6\sin 2\theta\cos 2\theta = 0$

$\hookrightarrow \sin 2\theta(1 - 6\cos 2\theta) = 0$

$\sin 2\theta = 0$

$2\theta = 0$ × invalid

$2\theta = 180$

$2\theta = 360$

$\Downarrow$

$\theta = 90$

$\theta = 180$

$\cos 2\theta = \frac{1}{6}$

$2\theta = 80.4059\ldots$

$2\theta = 279.5940\ldots$ $(360 - 80.4059)$

$\Downarrow$

$\theta = 40.2029\ldots$

$\theta = 139.7970\ldots$

$\boxed{\theta = 40.2°, 90°, 139.8°, 180°}$

(Total for question 4 is 8 marks)

5 Evaluate the following sum.
You must show your working.

$$\sum_{r=1}^{80}\left(240 - \frac{3r}{2}\right)$$

$$U_n = 240 - \frac{3r}{2}$$

$$a = U_1 = 240 - \frac{3(1)}{2} = 238.5$$

$$L = U_{80} = 240 - \frac{3(80)}{2} = 120$$

$$S_n = \frac{n}{2}(a + L)$$

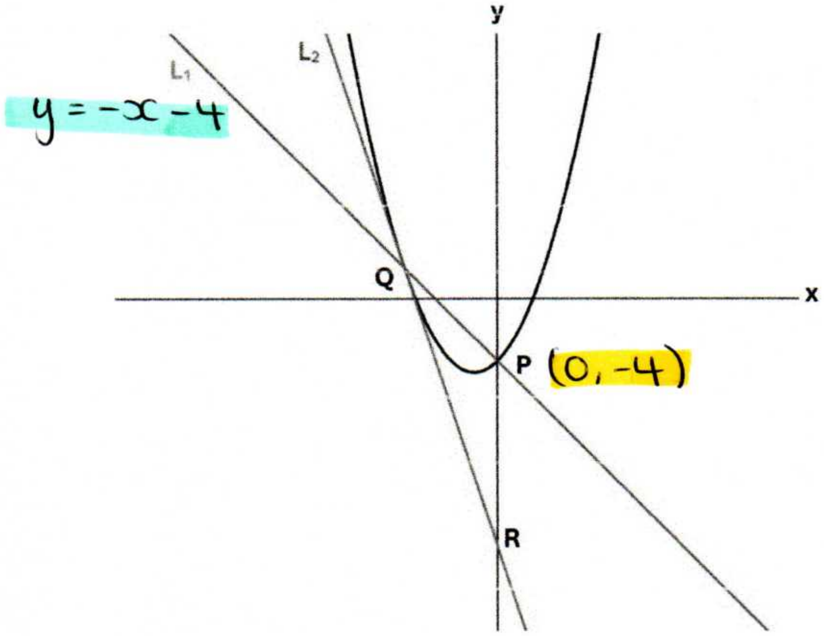
$$S_{80} = \frac{80}{2}(238.5 + 120)$$

$$S_{80} = 14340$$

14340

(Total for question 5 is 4 marks)

6 The figure below shows the curve C with equation $y = \frac{1}{3}x^2 + x - 4$



The curve crosses the y axis at point P. The normal to C at point P is the straight line L_1.

(a) Find an equation for line L_1 in the form y = mx + c

$y = \frac{1}{3}x^2 + x - 4$

$\frac{dy}{dx} = \frac{2}{3}x + 1$

tangent and normal are perpendicular so their gradients are the negative reciprocal

At point P, $x = 0$

gradient of tangent at P $= \frac{2}{3}(0) + 1 = 1$

gradient of normal at P $= -1$

$y - y_1 = m(x - x_1)$

$y - (-4) = -1(x - 0)$

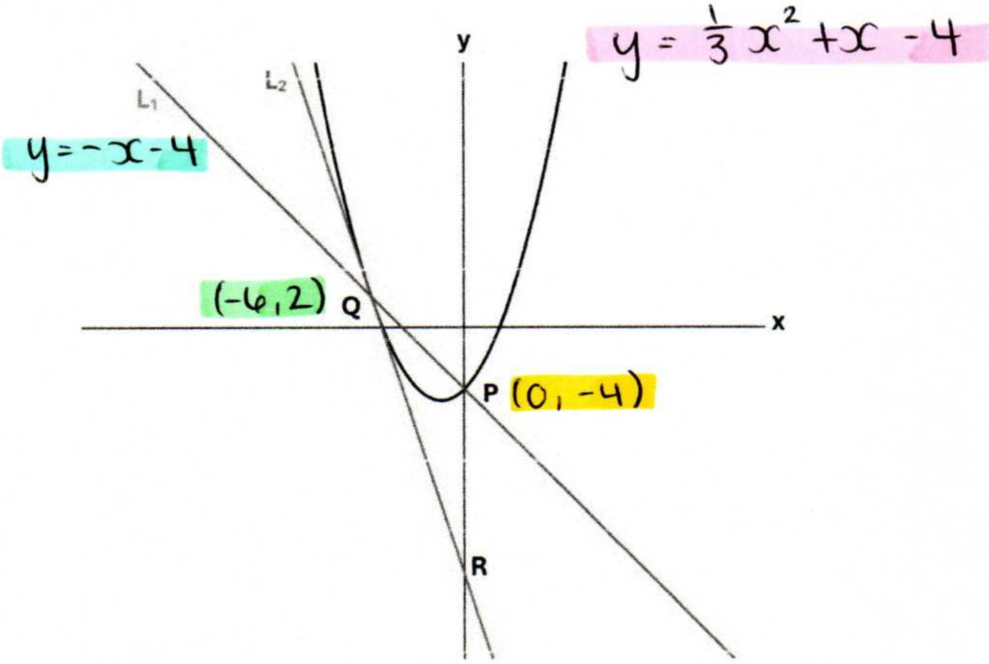
$y + 4 = -x$

$y = -x - 4$

$\boxed{y = -x - 4}$

(5)

6 (continued)



(b) The line L_1 meets the curve C again at point Q.
Determine the coordinates of point Q.

$$\frac{1}{3}x^2 + x - 4 = -x - 4$$

$$\frac{1}{3}x^2 + 2x = 0$$

×3

$$x^2 + 6x = 0$$

$$x(x + 6) = 0$$

$x = 0$ $\quad$ $x = -6$

$$y = -x - 4$$

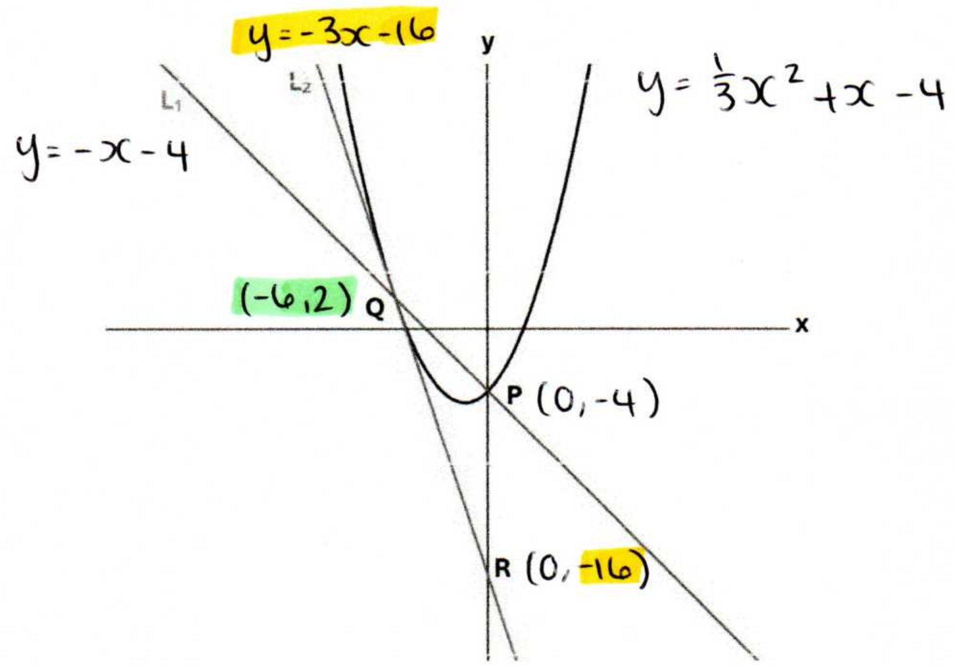
$$y = -(-6) - 4$$

$$y = 2$$

Q(-6 , 2)

(4)

6 (continued)



(c) The tangent to curve C at point Q is the straight line L_2

L_2 crosses the y axis at point R

Calculate the area of triangle PQR.

$$\frac{dy}{dx} = \frac{2}{3}x + 1$$

At point Q, $x = -6$

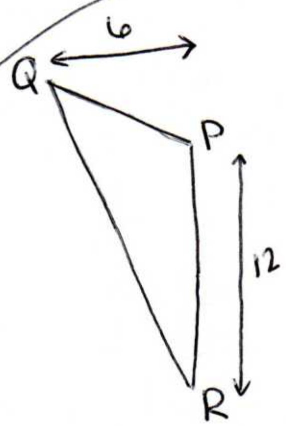
gradient of tangent at Q $= \frac{2}{3}(-6) + 1 = -3$

$y - y_1 = m(x - x_1)$

$y - 2 = -3(x - -6)$

$y - 2 = -3x - 18$

$y = -3x - 16$



$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

$$= \frac{12 \times 6}{2}$$

$$= 36$$

36 units^2

(6)

(Total for question 6 is 15 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS