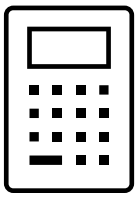
Surname:	Forename(s):

GCSE Mathematics

Completing the Square

ANSWERS

Grade 8



These questions get more difficult as your go through this paper.

This topic could come up on a calculator paper and non-calculator paper.

Materials

- **Formulae Sheet** (on the next page)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- Protractor
- Compass
- Tracing Paper
- **Calculator allowed**

Instructions

- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.

Higher Tier Formulae Sheet

Perimeter, Area and Volume

Where a and b are the lengths of the parallel sides and h Is their perpendicular separation:

Area of a trapezium = ½ (a + b) h

Volume of a prism = area of cross section x length

Where r is the radius and d is the diameter:

Circumference of a circle = 2πr = πd

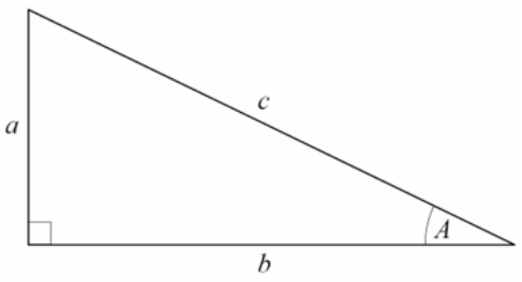
Area of a circle = πr^2

Quadratic Formula

The solution of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

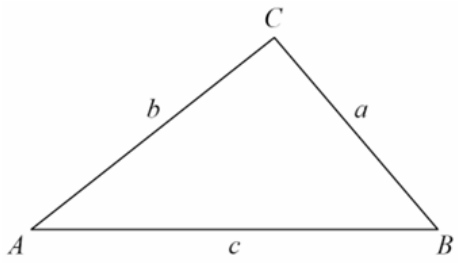


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle } = \frac{1}{2} ab \sin C$$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A

and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

1 Write down the coordinates for the turning point of the graph $y = (x + 3)^2 + 9$

$y = (x+3)^2 + 9$

$x+3 = 0$ $\quad$ $y = 9$

$x = -3$

(−3 , 9)

(Total for question 1 is 1 mark)

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2 Write down the coordinates for the turning point of the graph $y = (x + 7)^2 - 8$

$y = (x+7)^2 - 8$

$x+7 = 0$ $\quad$ $y = -8$

$x = -7$

(−7 , −8)

(Total for question 2 is 1 mark)

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3 Write down the coordinates for the turning point of the graph $y = (x - 2)^2 + 2$

$y = (x-2)^2 + 2$

$x-2 = 0$ $\quad$ $y = 2$

$x = 2$

(2 , 2)

(Total for question 3 is 1 mark)

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4 Write down the coordinates for the turning point of the graph $y = (x - 5)^2 - 4.25$

$y = (x-5)^2 - 4.25$

$x-5 = 0$ $\quad$ $y = -4.25$

$x = 5$

(5 , −4.25)

(Total for question 4 is 1 mark)

5 Write down the coordinates for the turning point of the graph $y = 2(x - 3)^2 + 6$

$y = 2(x-3)^2 + 6$

$x - 3 = 0$ $\quad$ $y = 6$

$x = 3$

(3 , 6)

(Total for question 5 is 1 mark)

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6 Write down the coordinates for the turning point of the graph $y = \frac{2}{3}(x + 6)^2 + 9$

$y = \frac{2}{3}(x+6)^2 + 9$

$x + 6 = 0$ $\quad$ $y = 9$

$x = -6$

(-6 , 9)

(Total for question 6 is 1 mark)

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7 (a) Write $x^2 + 4x - 5$ in the form $(x + a)^2 + b$, where a and b are integers.

$x^2 + 4x - 5$

$(x+2)^2 - (2)^2 - 5$

$(x+2)^2 - 4 - 5$

$(x+2)^2 - 9$

$(x+2)^2 - 9$

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph $y = x^2 + 4x - 5$

$y = (x+2)^2 - 9$

$x + 2 = 0$ $\quad$ $y = -9$

$x = -2$

(-2 , -9)

(1)

(Total for question 7 is 3 marks)

8 (a) Write $x^2 - 10x + 2$ in the form $(x + a)^2 + b$, where a and b are integers.

$x^2 - 10x + 2$

$(x-5)^2 - (-5)^2 + 2$

$(x-5)^2 - 25 + 2$

$(x-5)^2 - 23$

$(x-5)^2 - 23$

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = x^2 - 10x + 2$

$y = (x-5)^2 - 23$

$x - 5 = 0$ $\quad$ $y = -23$

$x = 5$

(5 , -23)

(1)

(Total for question 8 is 3 marks)

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9 (a) Write $2x^2 + 8x - 3$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

$2x^2 + 8x - 3$

$2[x^2 + 4x] - 3$

$2[(x+2)^2 - (2)^2] - 3$

$2[(x+2)^2 - 4] - 3$

$2(x+2)^2 - 8 - 3$

$2(x+2)^2 - 11$

$2(x+2)^2 - 11$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 2x^2 + 8x - 3$

$y = 2(x+2)^2 - 11$

$x + 2 = 0$ $\quad$ $y = -11$

$x = -2$

(-2 , -11)

(1)

(Total for question 9 is 4 marks)

10 (a) Write $3x^2 - 6x - 7$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

$3x^2 - 6x - 7$

$3[x^2 - 2x] - 7$

$3[(x-1)^2 - (1)^2] - 7$

$3[(x-1)^2 - 1] - 7$

$3(x-1)^2 - 3 - 7$

$3(x-1)^2 - 10$

$3(x-1)^2 - 10$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 3x^2 - 6x - 7$

$y = 3(x-1)^2 - 10$

$x - 1 = 0$ $\quad$ $y = -10$

$x = 1$

(1 , -10)

(1)

(Total for question 10 is 4 marks)

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11 (a) Write $2x^2 + 10x + 4$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

$2x^2 + 10x + 4$

$2[x^2 + 5x] + 4$

$2[(x + \frac{5}{2})^2 - (\frac{5}{2})^2] + 4$

$2[(x + \frac{5}{2})^2 - \frac{25}{4}] + 4$

$2(x + \frac{5}{2})^2 - \frac{25}{2} + 4$

$2(x + \frac{5}{2})^2 - \frac{17}{2}$

$2(x + \frac{5}{2})^2 - \frac{17}{2}$

OR $2(x+2.5)^2 - 8.5$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 2x^2 + 10x + 4$

$y = 2(x + \frac{5}{2})^2 - \frac{17}{2}$

$x + \frac{5}{2} = 0$ $\quad$ $y = -\frac{17}{2}$

$x = -\frac{5}{2}$

$(-\frac{5}{2}, -\frac{17}{2})$

OR $(-2.5, -8.5)$ (1)

(Total for question 11 is 4 marks)

12 (a) Write $3x^2 - 4x + 10$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

$3x^2 - 4x + 10$

$3\left[x^2 - \frac{4}{3}x\right] + 10$

$3\left[\left(x - \frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right] + 10$

$3\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9}\right] + 10$

$3\left(x - \frac{2}{3}\right)^2 - \frac{12}{9} + 10$

$3\left(x - \frac{2}{3}\right)^2 + \frac{26}{3}$

$3\left(x - \frac{2}{3}\right)^2 + \frac{26}{3}$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 3x^2 - 4x + 10$

$y = 3\left(x - \frac{2}{3}\right)^2 + \frac{26}{3}$

$x - \frac{2}{3} = 0$ $\quad y = \frac{26}{3}$

$x = \frac{2}{3}$

$\left(\frac{2}{3}, \frac{26}{3}\right)$

(1)

(Total for question 12 is 4 marks)

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13 (a) Write $7x^2 + 5x - 4$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

$7x^2 + 5x - 4$

$7\left[x^2 + \frac{5}{7}x\right] - 4$

$7\left[\left(x + \frac{5}{14}\right)^2 - \left(\frac{5}{14}\right)^2\right] - 4$

$7\left[\left(x + \frac{5}{14}\right)^2 - \frac{25}{196}\right] - 4$

$7\left(x + \frac{5}{14}\right)^2 - \frac{25}{28} - 4$

$7\left(x + \frac{5}{14}\right)^2 - \frac{137}{28}$

$7\left(x + \frac{5}{14}\right)^2 - \frac{137}{28}$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 7x^2 + 5x - 4$

$y = 7\left(x + \frac{5}{14}\right)^2 - \frac{137}{28}$

$x + \frac{5}{14} = 0$ $\quad y = -\frac{137}{28}$

$x = -\frac{5}{14}$

$\left(-\frac{5}{14}, -\frac{137}{28}\right)$

(1)

(Total for question 13 is 4 marks)

14 By completing the square, solve $x^2 + 4x + 3 = 0$

$x^2+4x+3=0$

$(x+2)^2-(2)^2+3=0$

$(x+2)^2-4+3=0$

$(x+2)^2-1=0$

(+1) (+1)

$(x+2)^2=1$

(√) (√)

$x+2=\pm 1$

(-2) (-2)

$x=-2\pm 1$

$x=-2+1 \quad x=-2-1$

$=-1 \quad =-3$

x = -1 or -3

(Total for question 14 is 5 marks)

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15 By completing the square, solve $x^2 - 8x + 5 = 0$

Give your answer in surd form.

$x^2-8x+5=0$

$(x-4)^2-(4)^2+5=0$

$(x-4)^2-16+5=0$

$(x-4)^2-11=0$

(+11) (+11)

$(x-4)^2=11$

(√) (√)

$x-4=\pm\sqrt{11}$

(+4) (+4)

$x=4\pm\sqrt{11}$

$x = 4\pm\sqrt{11}$

(Total for question 15 is 5 marks)