Surname: | Forename(s):

A Level Mathematics – Pure Maths

Paper 2

Practice Set A

MyStudy success

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n - 1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\,\mathrm{cosec}^2\, kx$
$\mathrm{cosec}\, kx$	$-k\,\mathrm{cosec}\, kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

Integration	**(+ constant)**
$f(x)$	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\mathrm{cosec}\, kx$	$-\frac{1}{k}\ln\lvert\mathrm{cosec}\, kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

A Level (Pure) Formulae Sheet continued

Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

where θ is measured in radians

1 The polynomial f(x) is defined as

$$f(x) = 3x^3 - 13x^2 + 18x - 8$$

(a) Using factor theorem, show that (x – 1) is a factor of f(x).

(2)

(b) Express f(x) as a product of its three linear factors.

(2)

(Question 1 continued)

The polynomial f(x) is defined as

$$f(x) = 3x^3 – 13x^2 + 18x - 8$$

(c) Find the remainder when f(x) is divided by (x + 1)

(1)

(d) Determine the value of the constants a, b and c so that

$$f(x) = (x + 1) (3x^2 + ax + b) + c$$

a = ______________

b = ______________

c = ______________

(3)

(Total for question 1 is 8 marks)

2 An area development scheme aims to optimise office workspace in a city.
The scheme is planned to start in 2025 and finish in 2042.

The scheme plans to build 325 office spaces in the year 2030.
The scheme plans to build 205 office spaces in the year 2036.

The number of office spaces that the area development scheme plan to build each year forms an arithmetic sequence.

(a) How many office spaces are planned to be developed in the first year of this scheme?

____

(3)

(b) How many office spaces are planned to be developed in total over the duration of this scheme?

____

(3)

(Total for question 2 is 6 marks)

3 Express $\frac{5x^2 + 10x + 3}{(x-1)(x+2)^2}$ in the form $\frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$

Where A,B and C are constant to be found.

(Total for question 3 is 4 marks)

4 Solve for $0 < \theta < 2\pi$

$$3\cos(2\theta-\pi) = 8\tan(2\theta-\pi)$$

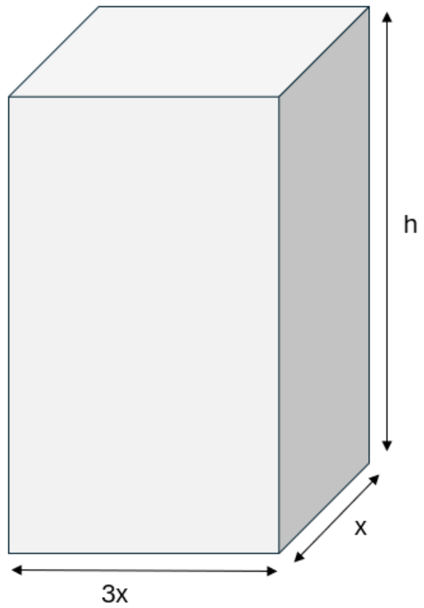
Give your answers in radians to 2 decimal places.

(Total for question 4 is 8 marks)

5 Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 6 for all positive integer values of n.

(Total for question 5 is 4 marks)

6 The figure below shows a solid cuboid. All measurements are in centimetres.



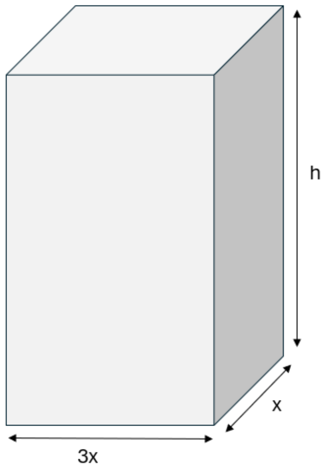
The total surface area of the cuboid is 640 cm^2

(a) Show that that volume, V cm^3, of the cuboid can be written as

$$V = 240x - \frac{9}{4}x^3$$

(4)

(Question 6 continued



(b) Find the value of x for which V is stationary. Give your answer as an exact value

x = ____________________

(4)

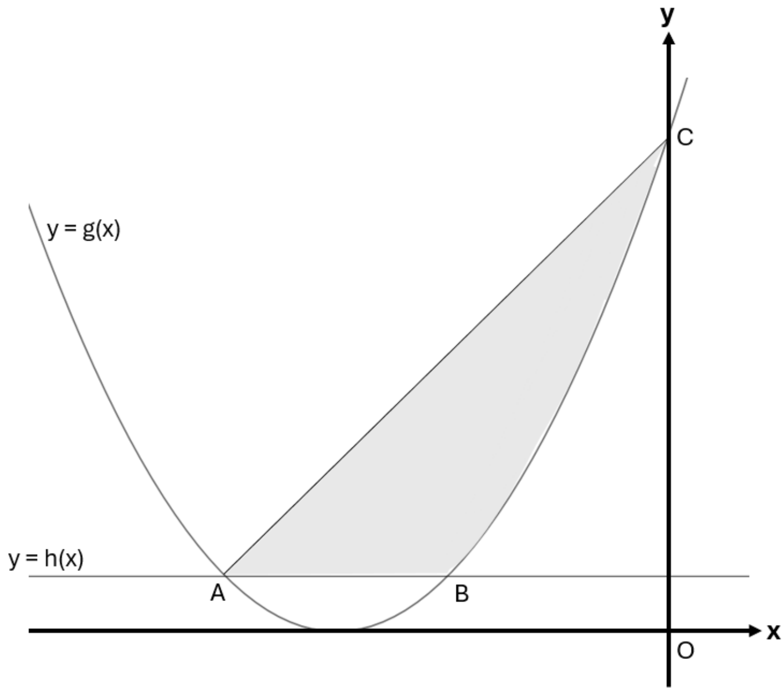
(c) Calculate the maximum value for V, justifying why this value is a maximum value.

(4)

(Total for question 6 is 12 marks)

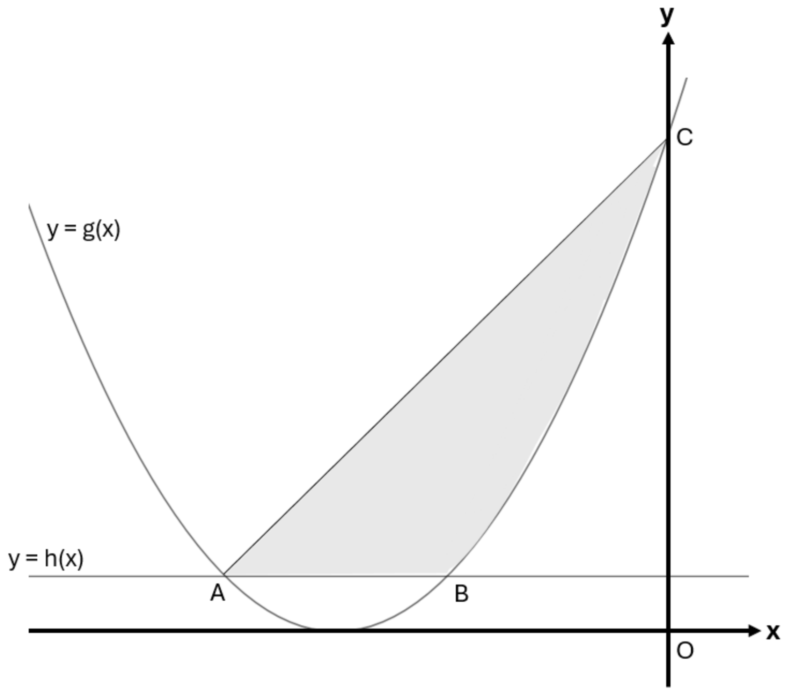
7 The diagram shows curve with equation $g(x) = (-\frac{1}{2}x - 3)^2$

and straight line with equation $h(x) = 1$ $\qquad x \in \mathbb{R}$



g(x) intersects h(x) at points A and B.
g(x) meets the y axis at point C
AC is a straight line.
Calculate the exact area of the shaded region bound by the curve, the straight line AB and the straight line AC.

(Question 7 continued)



Area = ______________________ units^2

(Total for question 7 is 8 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS