Surname:	Forename(s):

A Level Mathematics – Pure Maths

Paper 1

Practice Set A

ANSWERS

/50

Materials

- **Formulae Sheet** (on the next pages)
- **Black** ink or ball-point pen
- Pencil
- Eraser
- If needed (protractor, compass, tracing paper)
- Spare plain paper
- **Calculator allowed**

Instructions

- Time allowed: **60 minutes.**
- The total mark for this paper is **50**.
- Show **all** of your working.
- Draw diagrams in pencil.
- Plot graphs in pencil.
- Diagrams are **not** accurately drawn, unless otherwise indicated.
- Cross through any work you don't want to be marked.
- Use plain paper for extra working out space if needed.

A Level (Pure) Formulae Sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic Sequence

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n - 1)d]$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric Sequence

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n - r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\,\mathrm{cosec}^2\, kx$
$\mathrm{cosec}\, kx$	$-k\,\mathrm{cosec}\, kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

Integration	**(+ constant)**
$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\mathrm{cosec}\, kx$	$-\frac{1}{k}\ln\lvert\mathrm{cosec}\, kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

A Level (Pure) Formulae Sheet continued

Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

where θ is measured in radians

1 Simplify

(a) $(\sqrt{5}+1)^2$

$(\sqrt{5}+1)(\sqrt{5}+1)$

$5+\sqrt{5}+\sqrt{5}+1$

$6+2\sqrt{5}$

$6+2\sqrt{5}$

(2)

(b) $\frac{3+\sqrt{3}}{3-\sqrt{3}}+2\sqrt{48}-(\sqrt{2})^2$

$\frac{(3+\sqrt{3})}{(3-\sqrt{3})}\times\frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$\hookrightarrow \frac{9+3\sqrt{3}+3\sqrt{3}+3}{9+\cancel{3\sqrt{3}}-\cancel{3\sqrt{3}}-3}$

$\hookrightarrow \frac{12+6\sqrt{3}}{6}$

$\hookrightarrow 2+\sqrt{3}$

$2\sqrt{48}$

$\hookrightarrow 2\sqrt{16}\sqrt{3}$

$\hookrightarrow 2\times4\sqrt{3}$

$\hookrightarrow 8\sqrt{3}$

$(\sqrt{2})^2$

$\hookrightarrow \sqrt{2}\times\sqrt{2}$

$\hookrightarrow 2$

$\cancel{2}+\sqrt{3}+8\sqrt{3}-\cancel{2}$

$9\sqrt{3}$

$9\sqrt{3}$

(6)

(Total for question 1 is 8 marks)

2 Solve the simultaneous equation

$$9 = x + y \rightarrow y = 9 - x$$

$$x^2 - 3xy + 2y^2 = 0$$

$$x^2 - 3x(9-x) + 2(9-x)^2 = 0$$

$$x^2 - 3x(9-x) + 2(9-x)(9-x) = 0$$

$$x^2 - 27x + 3x^2 + 2(81 - 18x + x^2) = 0$$

$$x^2 - 27x + 3x^2 + 162 - 36x + 2x^2 = 0$$

$$6x^2 - 63x + 162 = 0$$

$$(6x - 27)(x - 6) = 0$$

$x = 4.5$ $\quad$ $x = 6$

$y = 9 - 4.5$ $\quad$ $y = 9 - 6$

$y = 4.5$ $\quad$ $y = 3$

$x = 4.5,\ y = 4.5$

OR $x = 6,\ y = 3$

(Total for question 2 is 6 marks)

3 Given that the equation $x^2 + (p - 3)x + (6 - p) = 0$ has two distinct real roots, find the values of p.

$\hookrightarrow b^2 - 4ac > 0$

$x^2 + (p-3)x + (6-p) = 0$

$a = 1$

$b = p - 3$

$c = 6 - p$

$b^2 - 4ac > 0$

$(p-3)(p-3) - 4(1)(6-p) > 0$

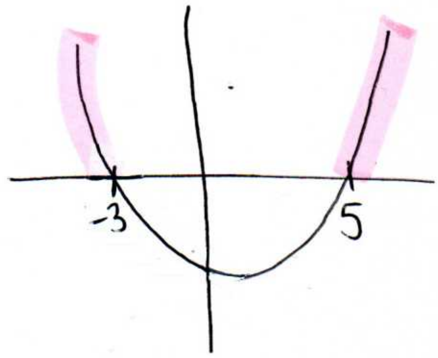
$p^2 - 3p - 3p + 9 - 24 + 4p > 0$

$p^2 - 2p - 15 > 0$

$(p-5)(p+3) > 0$

$\downarrow \qquad \downarrow$

$p = 5 \qquad p = -3$



$p < -3$ or $p > 5$

Set notation $\{p : p < -3\} \cup \{p : p > 5\}$

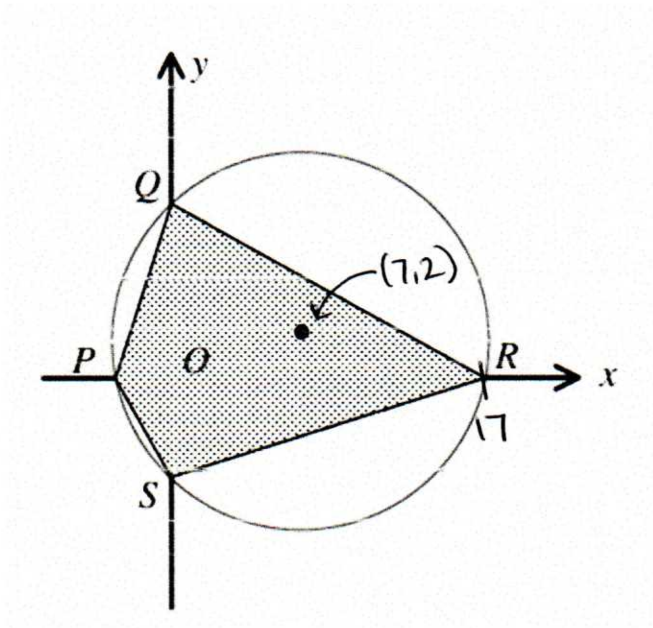
$\{p : p < -3\} \cup \{p : p > 5\}$

(Total for question 3 is 6 marks)

4 The diagram shows a circle with centre (7,2)

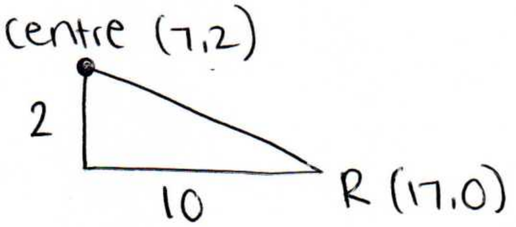
The circle intersects the x axis at points P and R.

The circle intersects the y axis at points Q and S.



(a) Given that R(17,0), find an equation of the circle.

Centre (7,2)



radius $= \sqrt{2^2 + 10^2}$

$= \sqrt{104}$

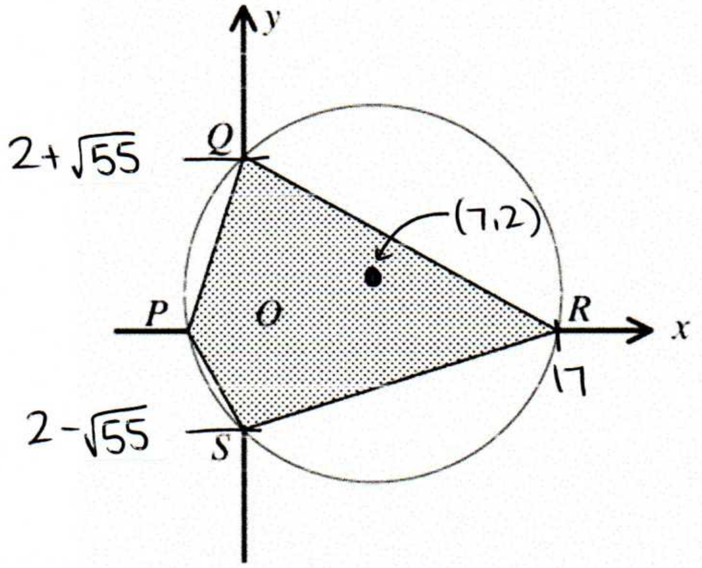
$(x+a)^2 + (y+b)^2 = r^2$

$(x-7)^2 + (y-2)^2 = 104$

$(x-7)^2 + (y-2)^2 = 104$

(4)

(Question 4 continued)



(b) Show that the length of QS = $2\sqrt{55}$

$$(x-7)^2+(y-2)^2=104$$
$$(y-2)^2=104-(x-7)^2$$
$$y-2=\pm\sqrt{104-(x-7)^2}$$
$$y=2\pm\sqrt{104-(x-7)^2}$$

when $x=0$, $y=2\pm\sqrt{104-(-7)^2}$

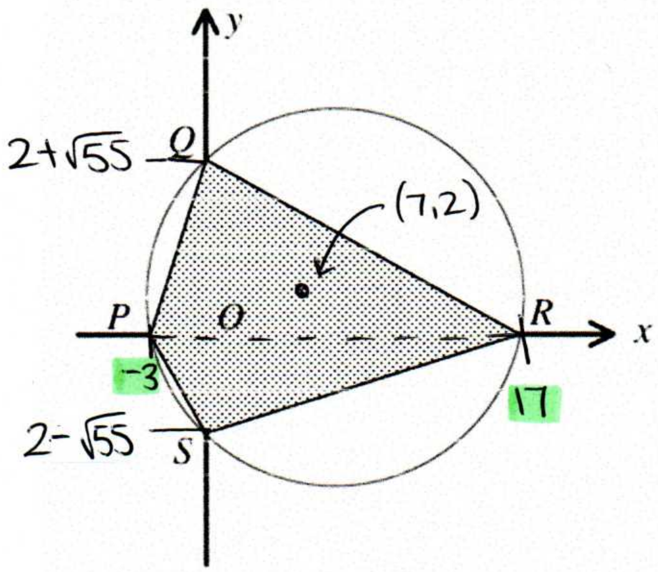
$y=2+\sqrt{55}$ or $2-\sqrt{55}$

$$QS = 2+\sqrt{55}-(2-\sqrt{55})$$
$$=\cancel{2}+\sqrt{55}-\cancel{2}+\sqrt{55}$$
$$=2\sqrt{55}$$

$2\sqrt{55}$

(4)

(Question 4 continued)



(c) Work out the area of quadrilateral PQRS.

$$(x-7)^2+(y-2)^2=104$$

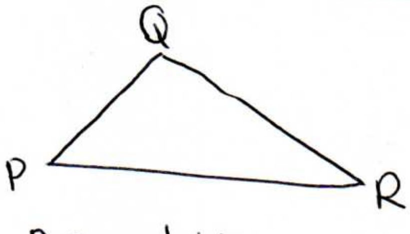
$$(x-7)^2=104-(y-2)^2$$

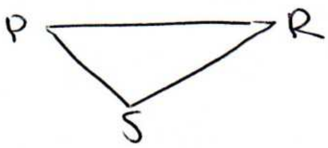
$$x-7=\pm\sqrt{104-(y-2)^2}$$

$$x=7\pm\sqrt{104-(y-2)^2}$$

when $y=0$, $x=7\pm\sqrt{104-(-2)^2}$

$x=17$ or -3





Area $=\frac{1}{2}bh$

$=\frac{1}{2}(20)(2+\sqrt{55})$

$=94.16198487\ldots$

Area $=\frac{1}{2}bh$

$=\frac{1}{2}(20)(2-\sqrt{55})$

$=|-54.16198487|$

$=54.16198487$

148.3 units^2

(5)

Area $=94.161\ldots+54.161\ldots=148.3239697\ldots$

(Total for question 4 is 13 marks)

5 The point P (8,56) lies on the curve C.

The gradient function of curve C is given by

$$\frac{dy}{dx} = 6\sqrt[3]{x} - 7 \qquad x \geq 0$$

Find an equation for curve C.

$$\frac{dy}{dx} = 6x^{\frac{1}{3}} - 7$$

$$\int 6x^{\frac{1}{3}} - 7 \, dx = \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} - 7x + c$$

$$y = \frac{9}{2}x^{\frac{4}{3}} - 7x + c$$

$$56 = \frac{9}{2}(8)^{\frac{4}{3}} - 7(8) + c$$

$$56 = 16 + c$$

$$-16 \qquad -16$$

$$40 = c$$

$$y = \frac{9}{2}x^{\frac{4}{3}} - 7x + 40$$

(Total for question 5 is 6 marks)

6 The first four terms of an arithmetic sequence are

u_1	u_2	u_3	u_4
3	(x – y)	(3x + y – 2)	(x – 5y)

Where x and y are constants to be found.

Show that the sum of the first 20 terms of this sequence is 820.

$u_2 - u_1 = u_3 - u_2$

$(x-y)-3 = (3x+y-2)-(x-y)$

$x-y-3 = 3x+y-2-x+y$

$x-y-3 = 2x+2y-2$

$0 = x+3y+1$

$u_2 - u_1 = u_4 - u_3$

$(x-y)-3 = (x-5y)-(3x+y-2)$

$x-y-3 = x-5y-3x-y+2$

$x-y-3 = -2x-6y+2$

$3x+5y-5=0$

$x+3y+1=0 \Rightarrow \times 3 \quad 3x+9y+3=0$

$3x+5y-5=0 \Rightarrow \times 1 \quad 3x+5y-5=0$

$-$

$4y+8=0$

$\hookrightarrow 4y=-8$

$\boxed{y=-2}$

$x+3y+1=0$

$x+3(-2)+1=0$

$\boxed{x=5}$

$u_1 = 3$

$u_2 = 5-(-2) = 7$

$u_3 = 3(5) + -2 - 2 = 11$

$u_4 = 5-5(-2) = 15$

$a=3$
$d=4$
$n=20$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{20} = \frac{20}{2}(2(3)+(20-1)4)$

(Total for question 6 is 11 marks)

END OF TEST

TOTAL FOR THIS PAPER IS 50 MARKS